Liquid Rocket Engine Sizing

LPL Crash Course Lecture Series

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Topics to Cover

- **Introduction**
- **Propellants**
- **Nozzle Sizing**
- **Chamber Sizing**
- **Jessie & James Example**

This lecture will be focused on application and minimal theory will be discussed

-Assuming Knowledge of basic thermodynamics, fluid dynamics, and converging/diverging nozzles

Ideal Rocket Equation

M = instantaneous mass of rocket
u = velocity of rocket
t = time
F = net force = thrust = $\dot{m}\ V_{eq}$
V_{eq} = equivalent engine exhaust velocity = Isp g_0

mf = full mass
me = empty mass
mp = mass of propellant
Isp = specific impulse

Newton's second law of motion: $\dfrac{d\,Mu}{dt} = F = V_{eq}\,\dfrac{d\,mp}{dt}$

$$M\,du + \cancel{u\,dM} = V_{eq}\,dmp$$

Assume we move with rocket --> u = 0

$$M\,du = -V_{eq}\,dM$$

Mass of rocket varies with time:

$M(t) = me + mp\,(t) \qquad dM = -d\,mp$

$$du = -V_{eq}\,\frac{dM}{M}$$

MR = propellant mass ratio = $\dfrac{mf}{me}$

$$\Delta u = -V_{eq}\ln(M)\Big|_{mf}^{me}$$

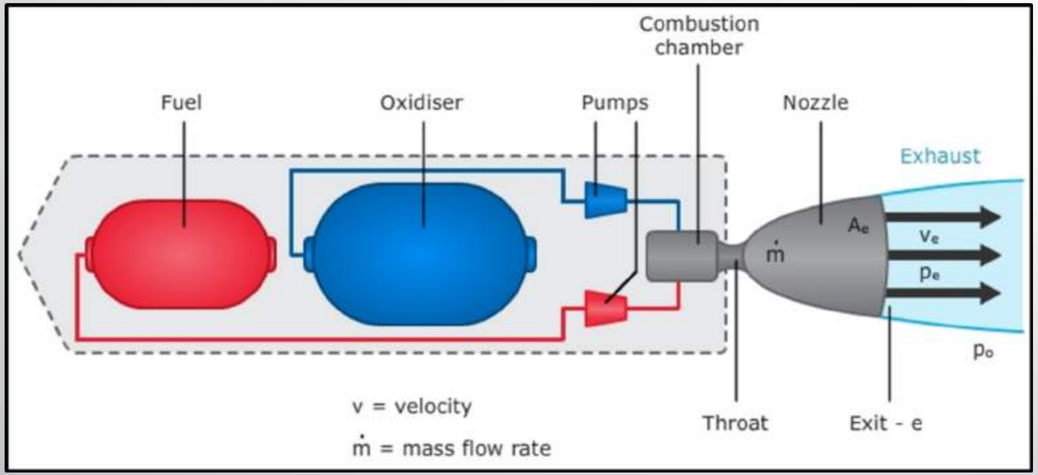
$$\boxed{\Delta u = V_{eq}\ln\left(\frac{mf}{me}\right) = V_{eq}\ln MR = Isp\ g_o\ln MR}$$

From https://spaceflightsystems.grc.nasa.gov/education/rocket/rktpow.html

Introduction

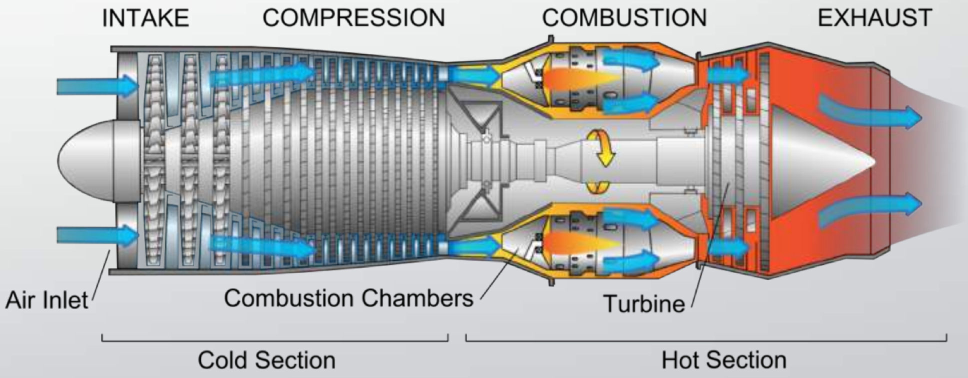
Difference between a rocket and jet engine?

Rocket Propulsion



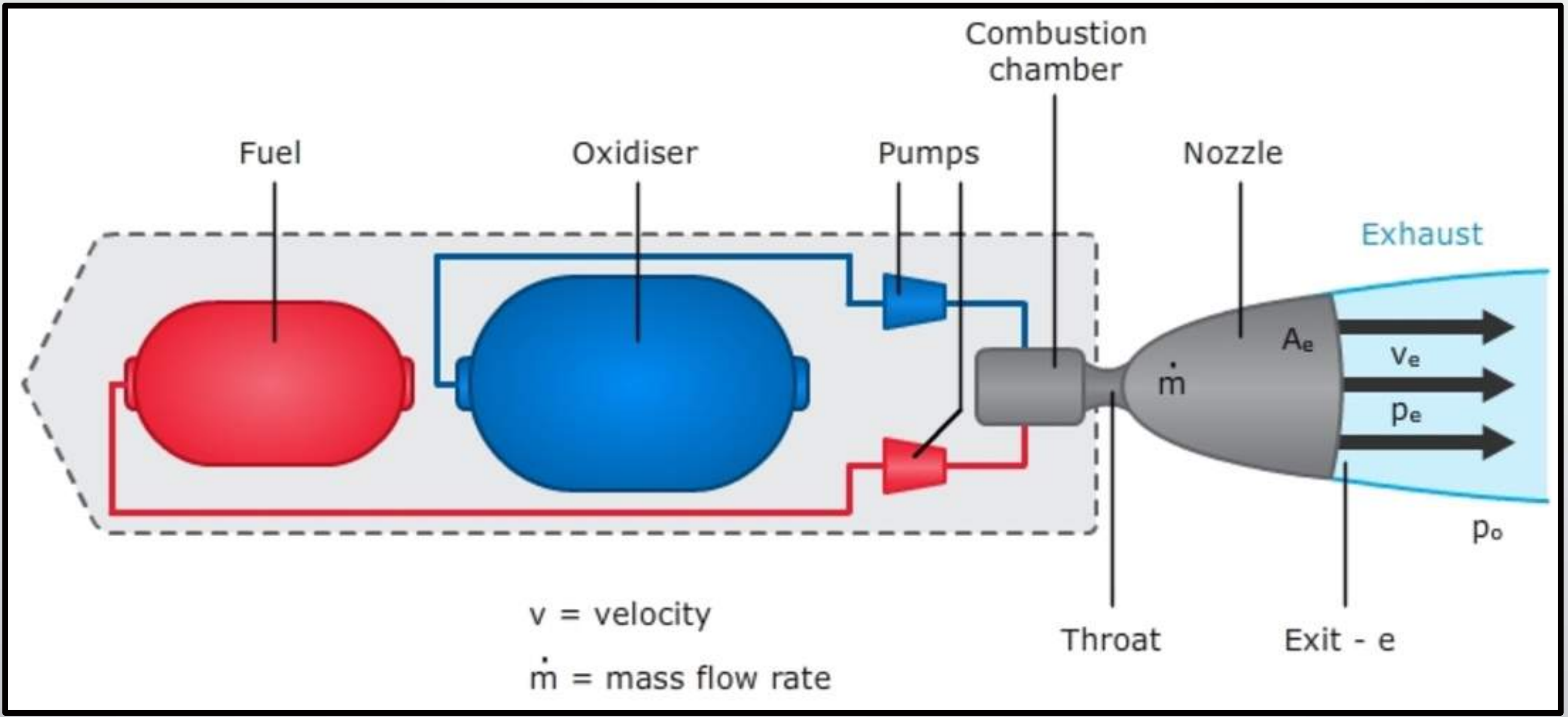
Rockets need to bring their full stock of propellant with them (Fuel and Oxidizer)

Air-Breathing Propulsion



Air-breathing propulsion pulls oxidizer in from the environment

Basic Rocket Architecture



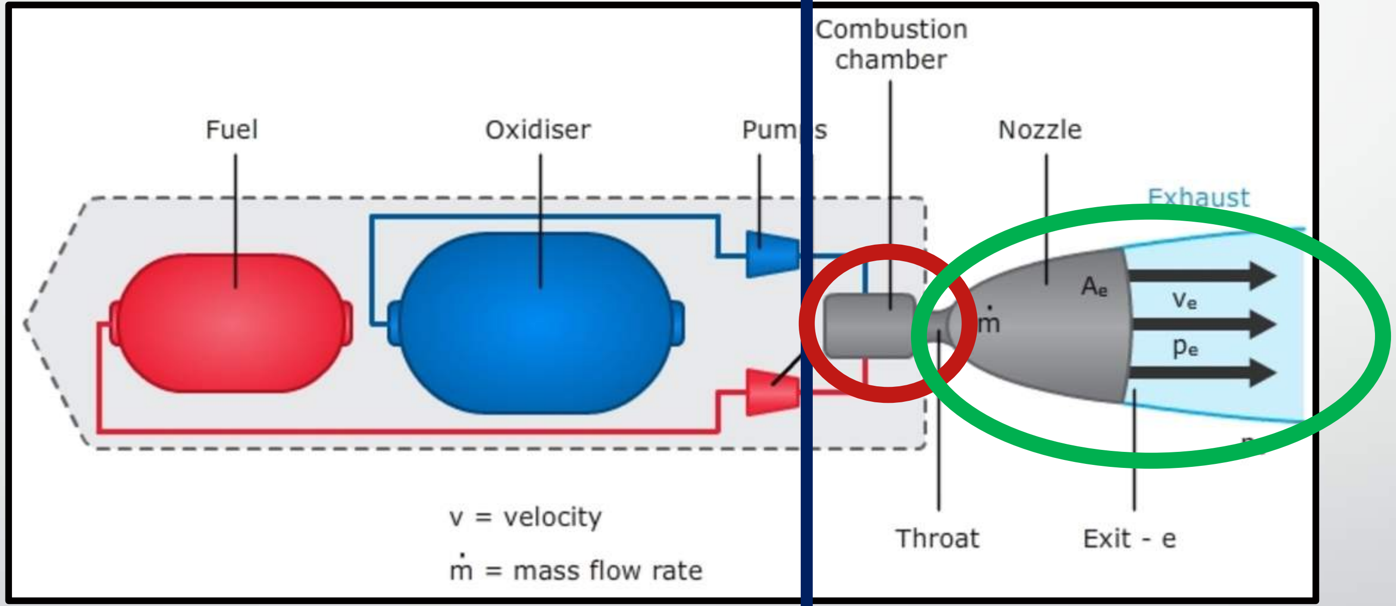
From https://www.grc.nasa.gov/www/k-12/rocket/lrockth.html

Thrust Generation

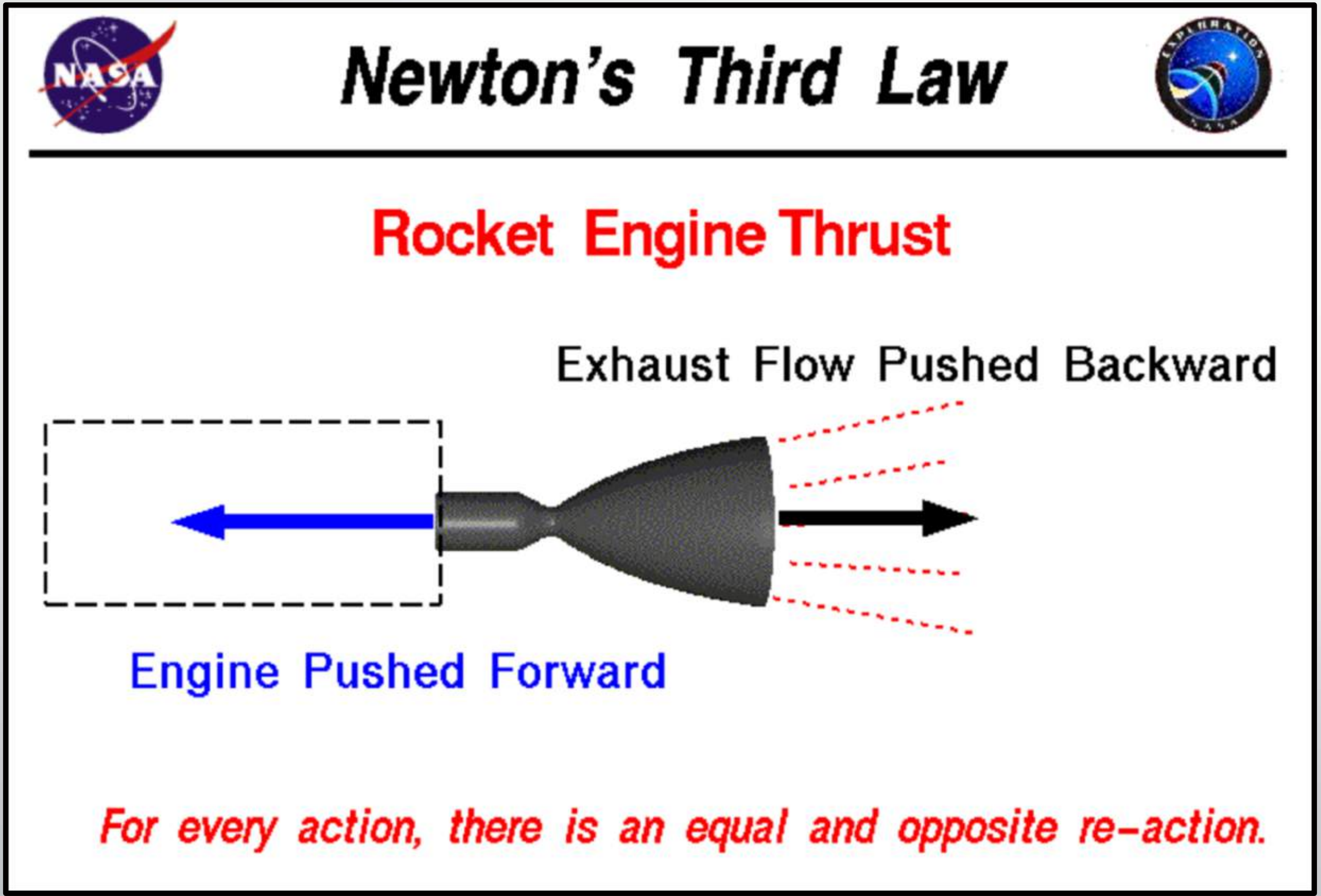
Potential Energy

Thermal Energy

Kinetic Energy



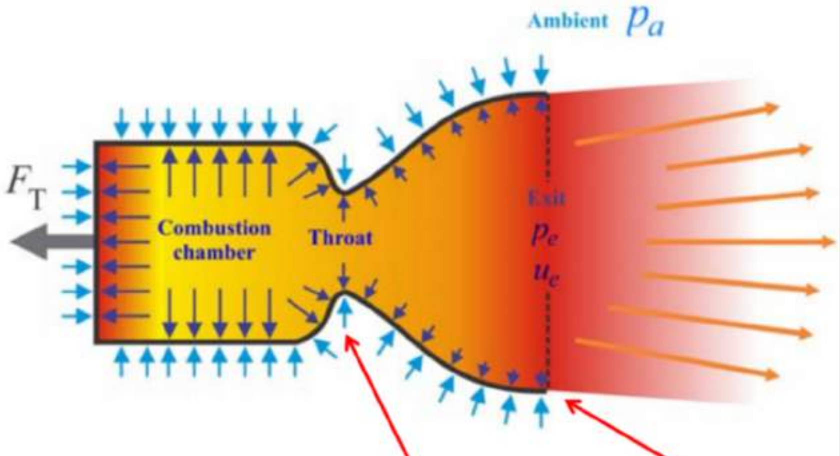
Rocket Engines generate thrust by taking potential energy (propellants), converting that to thermal energy (combustion chamber), and converting that into kinetic energy (nozzle)



From https://spaceflightsystems.grc.nasa.gov/education/rocket/newton3r.html

Thrust Equation

Typical Nozzle Expansion



Assumptions:

- working fluid/propellants expelled with the effective exhaust/exit velocity, u_e
- propellant flow rate, $\dot{m}$
- positive thrust in the direction opposite to u_e
- subscripts
  - e = exhaust, exit
  - a = ambient
- pressures p_e and p_a

Mach number	$M = 0$	$M_t = 1.0$	$M_e > 1$
pressure	$p = p_0$	$p_t \sim 0.5\ p_0$	$p_e < p_t$
temeprature	$T = T_0$	$T_t \sim (0.75 - 0.90) T_0$	$T_e < T_t$

$$\boxed{F_T = \dot{m}\, u_e + A_e\,(p_e - p_a)}$$

Courtesy of Professor Gruntman

$$\boxed{\underbrace{F_T}_{\text{Thrust}} = \underbrace{\dot{m}}_{\text{Mass Flow Rate}}\ \underbrace{u_e}_{\text{Exhaust Velocity}} + \underbrace{A_e}_{\text{Exit Area}}\,(\underbrace{p_e}_{\text{Exit Pressure}} - \underbrace{p_a}_{\text{Ambient Pressure}})}$$

Mach Number $M = \frac{u}{a}$

$a =$ speed of sound

$a = \sqrt{\gamma R T}$

Specific Impulse $I_{SP} = \frac{u_{eq}}{g}$

Specific Impulse (ISP)

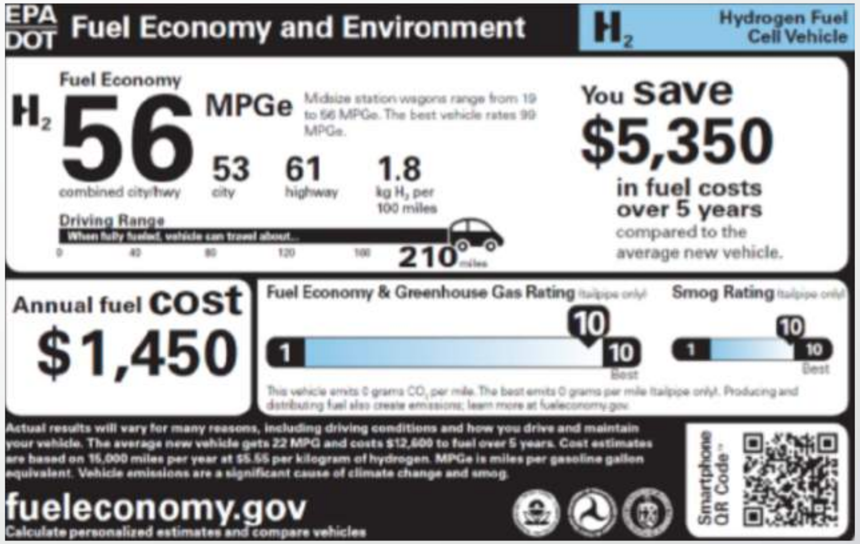
$$ISP = \frac{U_{eq}}{g_e} = \frac{F_t}{\dot{m} g_e}$$

$g_e = 9.81 \, \frac{m}{s^2}$

ISP [sec] is $\frac{Total\ Thrust}{Weight\ of\ Propellant}$

or

$$\frac{Thrust}{Weight\ of\ Propellant\ Consumed\ in\ one\ sec}$$



Think of ISP like you do for gas mileage for a car

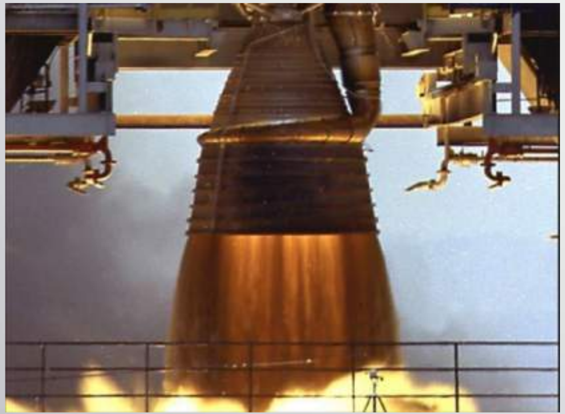
- How efficient is your conversion of propellant energy to spacecraft impulse?

Higher ISP the better rocket engine performance

- ISP is a function of chemistry

ISP of Various Propellants

Chemistry	ISP [sec]
Solids	220-300
Monopropellant	150-230
Liquid Hydrocarbon	250-350
Liquid Hydrogen / LOX	450



Saturn V F1 Engine
RP-1/LOX
ISP_{vac} = 300 sec
Exhaust Color: "Yellowish"

SpaceX Raptor Engine
Liquid Methane / LOX
ISP_{vac} = 360 sec
Exhaust Color: "Purplish"

Space Shuttle Main Engine
Liquid Hydrogen / LOX
ISP_{vac} = 450 sec
Exhaust Color: "Clear"

Design Choices

Which Propellants?

- *Not going to cover in this lecture*
- Considerations include:
  - Storability
    - Cryogenic?
    - Toxic?
    - Stability
  - Heat Transfer Properties
  - Density
  - Freezing Point
  - Contained Energy

Thermodynamics

Not covered fully in this lecture

- **Thermodynamics** will tell us:
    - Chamber temperature
    - Combustion gas specific heat
    - Combustion gas ratio of specific heats
    - Much more….

    - How do we practically determine thermodynamic properties at LPL?
        - NASA CEA

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NASA CEA

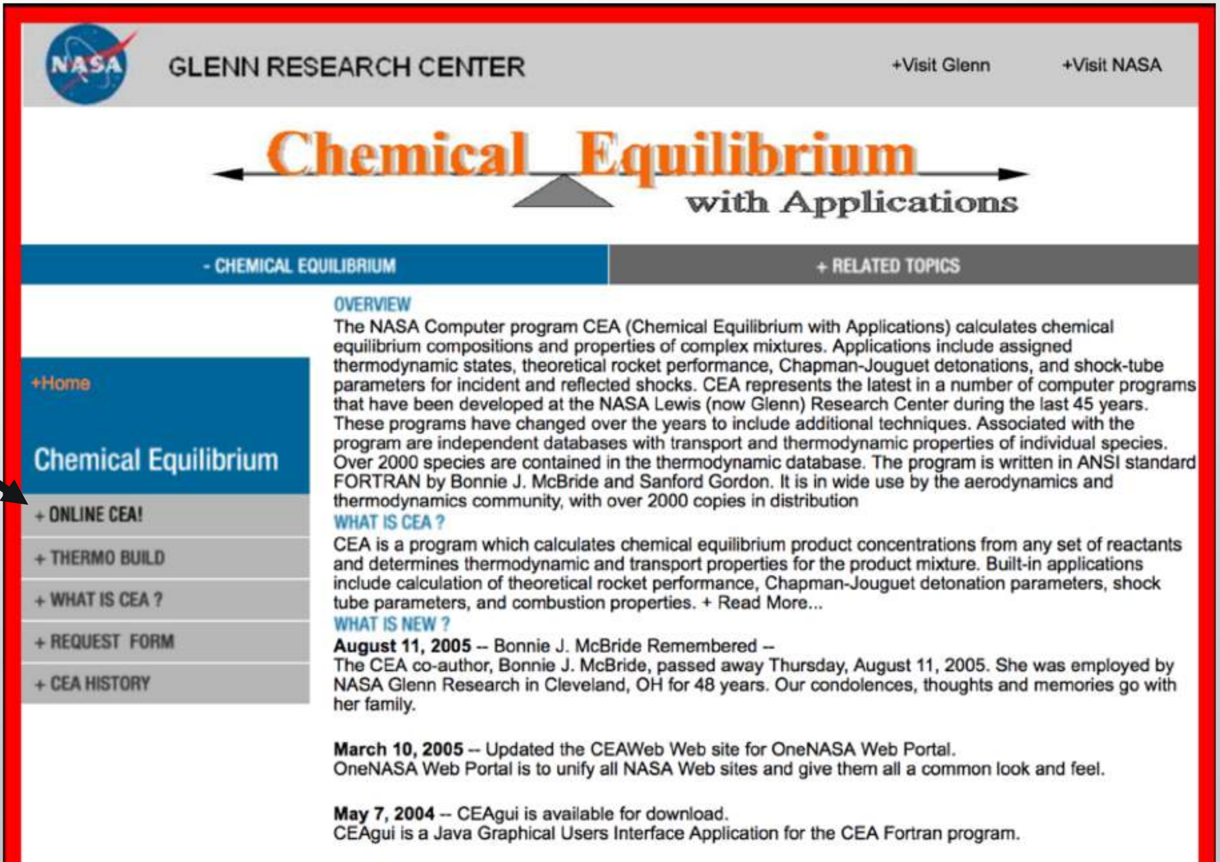
- Software used to determine thermodynamic properties of combustion gases
- Interpolates from lookup tables (that's why it's so quick!)

Online version

OR

Can download executable
(ask mike about running this)

https://www.grc.nasa.gov/www/CEAWeb/



NASA CEA

INPUTS

- Fuel
- Oxidizer
- Range of OF Ratios
- Range of Pressures

OUTPUTS

Temperature
Pressure
Density
Specific Heat
Gamma
Etc…

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          THERMODYNAMIC EQUILIBRIUM COMBUSTION PROPERTIES AT ASSIGNED

                                   PRESSURES

 CASE = LPLM0000

             REACTANT                    WT FRACTION      ENERGY      TEMP
                                          (SEE NOTE)     KJ/KG-MOL      K
 FUEL        RP-1                         1.0000000    -24717.700    298.150
 OXIDANT     O2                           1.0000000         0.000    298.150

 O/F=    1.50000  %FUEL= 40.000000  R,EQ.RATIO= 2.270444  PHI,EQ.RATIO= 2.270444

 THERMODYNAMIC PROPERTIES

 P, BAR            20.684   22.408   24.132   25.855   27.579   29.303   31.026   32.750
 T, K             2587.67  2588.74  2589.71  2590.59  2591.38  2592.11  2592.79  2593.41
 RHO, KG/CU M    1.6963 0 1.8371 0 1.9778 0 2.1186 0 2.2593 0 2.3999 0 2.5406 0 2.6812 0
 H, KJ/KG         -707.42  -707.42  -707.42  -707.42  -707.42  -707.42  -707.42  -707.42
 U, KJ/KG        -1926.77 -1927.16 -1927.52 -1927.84 -1928.13 -1928.40 -1928.64 -1928.87
 G, KJ/KG        -34305.5 -34221.8 -34143.9 -34071.0 -34002.5 -33937.9 -33876.7 -33818.6
 S, KJ/(KG)(K)    12.9839  12.9462  12.9113  12.8788  12.8484  12.8198  12.7929  12.7674

 M, (1/n)          17.645   17.647   17.648   17.649   17.651   17.652   17.653   17.654
 (dLV/dLP)t      -1.00132 -1.00128 -1.00124 -1.00120 -1.00116 -1.00113 -1.00111 -1.00108
 (dLV/dLT)p        1.0285   1.0275   1.0266   1.0258   1.0251   1.0244   1.0238   1.0232
 Cp, KJ/(KG)(K)    2.5461   2.5358   2.5265   2.5181   2.5104   2.5034   2.4970   2.4910
 GAMMAs            1.2414   1.2421   1.2427   1.2433   1.2438   1.2443   1.2447   1.2451
 SON VEL,M/SEC     1230.3   1230.9   1231.4   1231.8   1232.2   1232.6   1232.9   1233.2

 TRANSPORT PROPERTIES (GASES ONLY)
   CONDUCTIVITY IN UNITS OF MILLIWATTS/(CM)(K)

 VISC,MILLIPOISE  0.79165  0.79188  0.79209  0.79228  0.79245  0.79261  0.79275  0.79289

  WITH EQUILIBRIUM REACTIONS

 Cp, KJ/(KG)(K)    2.5461   2.5358   2.5265   2.5181   2.5104   2.5034   2.4970   2.4910
 CONDUCTIVITY      4.6452   4.6022   4.5636   4.5287   4.4969   4.4678   4.4411   4.4163
 PRANDTL NUMBER    0.4339   0.4363   0.4385   0.4405   0.4424   0.4441   0.4457   0.4472

  WITH FROZEN REACTIONS

 Cp, KJ/(KG)(K)    2.2417   2.2418   2.2419   2.2420   2.2420   2.2421   2.2421   2.2422
 CONDUCTIVITY      3.3889   3.3898   3.3907   3.3914   3.3921   3.3927   3.3932   3.3938
 PRANDTL NUMBER    0.5237   0.5237   0.5237   0.5238   0.5238   0.5238   0.5238   0.5238

 MASS FRACTIONS

 *CO             7.5920-1 7.5922-1 7.5923-1 7.5925-1 7.5926-1 7.5927-1 7.5928-1 7.5929-1
 *CO2            6.6677-2 6.6650-2 6.6626-2 6.6604-2 6.6584-2 6.6566-2 6.6549-2 6.6534-2
 *H              2.7558-4 2.6596-4 2.5731-4 2.4949-4 2.4237-4 2.3584-4 2.2984-4 2.2428-4
 HCO             9.7768-6 1.0208-5 1.0622-5 1.1023-5 1.1411-5 1.1786-5 1.2152-5 1.2507-5
 *H2             4.1152-2 4.1160-2 4.1166-2 4.1172-2 4.1178-2 4.1183-2 4.1188-2 4.1192-2
 H2O             1.3226-1 1.3228-1 1.3231-1 1.3233-1 1.3235-1 1.3236-1 1.3238-1 1.3240-1
 *OH             4.1911-4 4.0492-4 3.9216-4 3.8058-4 3.7003-4 3.6034-4 3.5141-4 3.4314-4
```

Multiple pages for multiple OF ratios

Exhaust Velocity

Exhaust Velocity

$$u_e = \sqrt{2\frac{\bar{R}\gamma}{\gamma-1}\frac{T_0}{\bar{M}}\left[1-\left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

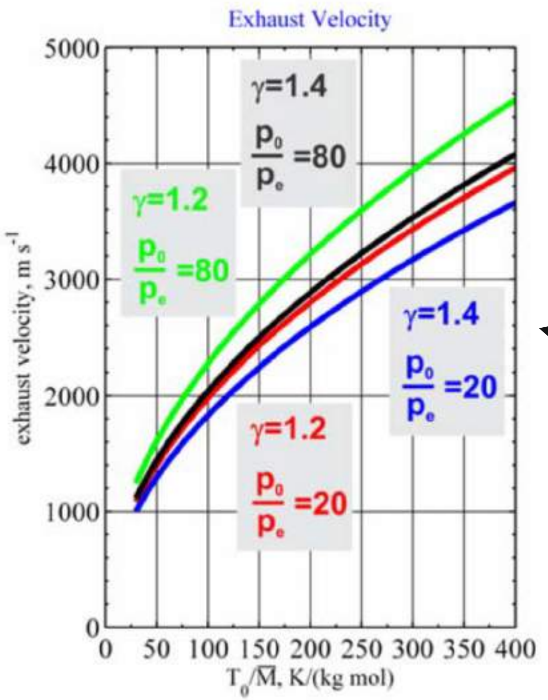
The higher $T_0/\bar{M}$ ratio

the higher u_e and I_{SP}

Maximum exhaust velocity

$$u_{e,MAX} = \sqrt{2\frac{\bar{R}\gamma}{\gamma-1}\frac{T_0}{\bar{M}}} = \sqrt{\frac{2\gamma RT_0}{\gamma-1}}$$

- at infinite pressure ratio
- temperature falls below liquefaction point
  - ➢ no longer a gas



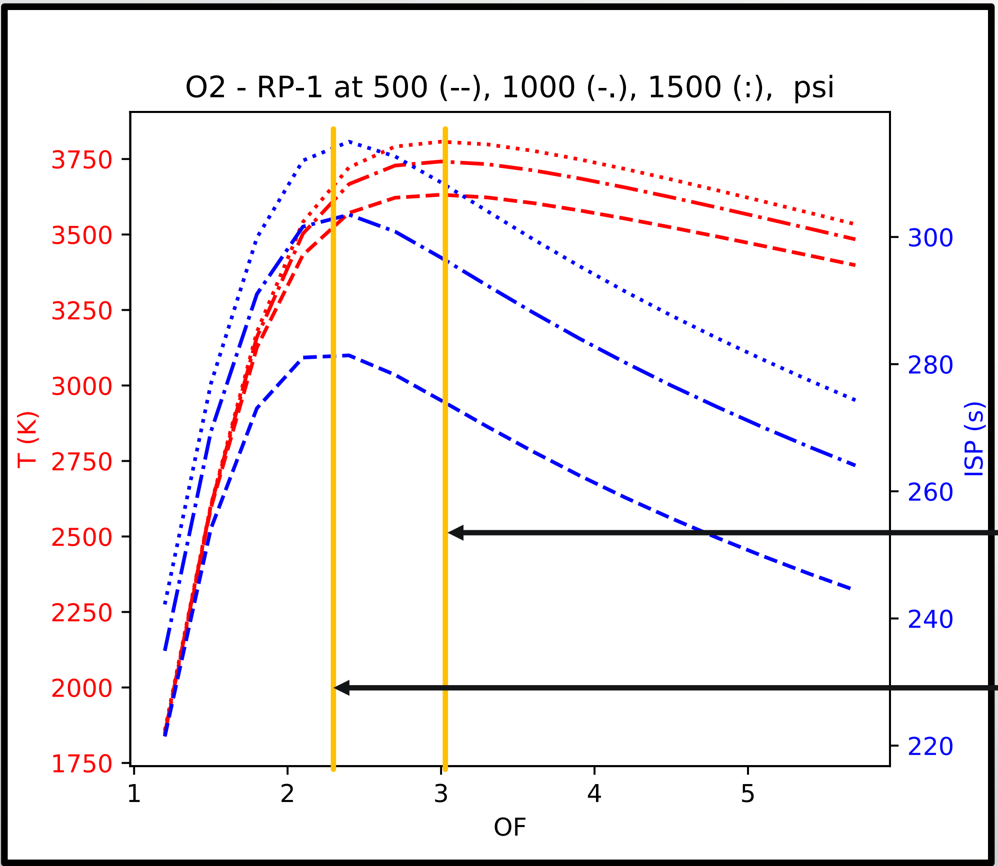
- decrease in γ results in increase of u_e
- increase in p_0 results in increase of u_e

Courtesy of Professor Gruntman

$$u_e = \sqrt{2\frac{\bar{R}\gamma}{\gamma-1}\frac{T_0}{\bar{M}}\left[1-\left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Want Higher u_e for more fuel efficient engine

CEA Results



Stoichiometric Ratio $\longrightarrow$ Highest Temperature

Highest $\frac{T_{\text{ch}}}{\bar{M}}$ $\longrightarrow$ Highest Fuel Efficiency

From LPL's CEA Execute & Read code

Design Choices

What Pressure?

Higher Pressure → Higher ISP
→ Higher Stress → More Engine Weight
→ More Feed System Weight
→ Higher Temperature → More Cooling

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Design Choices

What OF Ratio? *Usually fuel rich to some degree*

More Fuel Rich → Lower combustion product molecular weight → Higher ISP
→ Lower Temperature → Lower ISP
→ More Available Coolant

Trade-Off!

L P L

Nozzle Flow

Assumptions:

Quasi-1D Steady Isentropic Flow

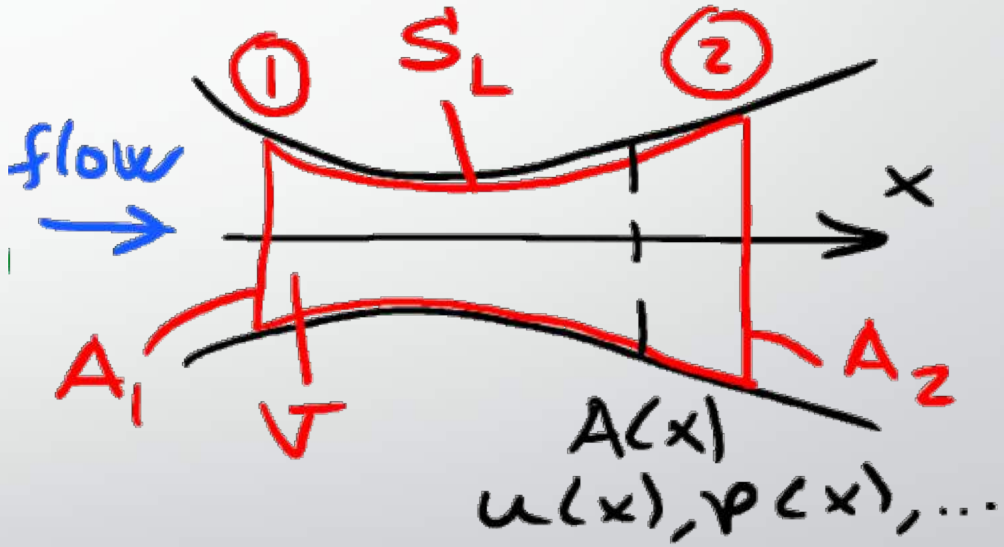
Quasi – Involves a slowly varying cross section

1D – flow properties are only changing in one dimension

Flow properties are uniform across each cross section

Steady – Flow is not changing in time

Isentropic – Flow is adiabatic and reversible

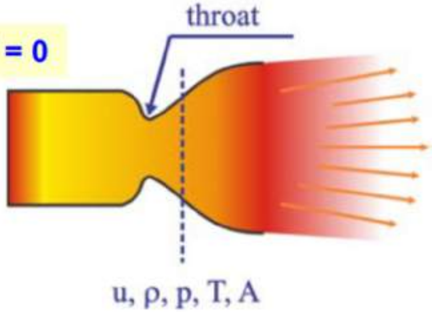


Nozzle Flow

Sonic Conditions

$M = 0$

p_0, T_0, ρ_0



Flow conditions at M = 1 are called **sonic conditions** (marked by asterisk, or star, or subscript "t" for throat)

$$\frac{\dot{m}}{A} = \frac{M\sqrt{\gamma}\,p_0}{\sqrt{RT_0}} \frac{1}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\frac{A}{A^*} = \frac{(\dot{m}/A^*)}{(\dot{m}/A)} = \frac{1}{M}\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

when M = 1

$$\frac{\dot{m}}{A^*} = \left(\frac{\dot{m}}{A}\right)_{M=1} = \frac{\sqrt{\gamma}\,p_0}{\sqrt{RT_0}}\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\boxed{\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

The Mach number is a function of the nozzle area

Courtesy of Professor Gruntman

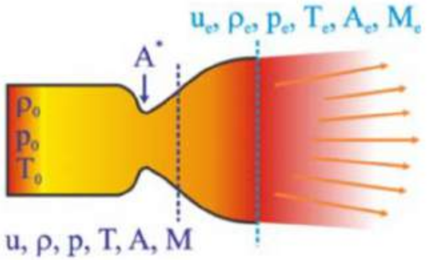
$$\boxed{\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Exit Conditions (e)

$$\boxed{\frac{A_e}{A^*} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}{M_e}^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Nozzle Flow

Nozzle Flow



$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

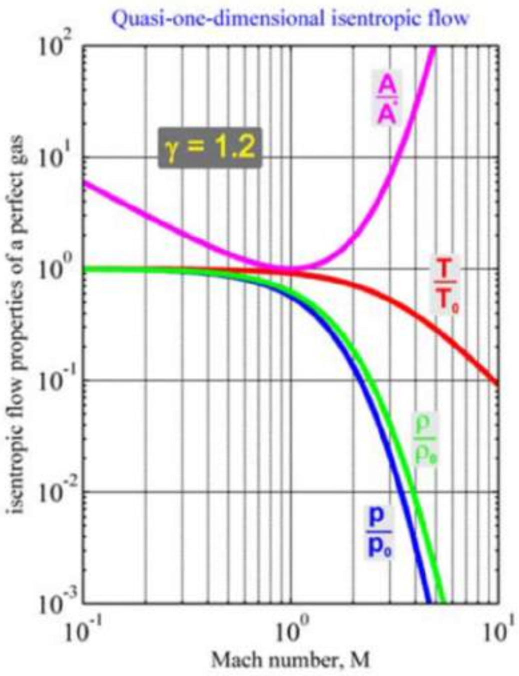
$$T = T_0\left[1+\frac{(\gamma-1)}{2}M^2\right]^{-1}$$

$$p = p_0\left[1+\frac{(\gamma-1)}{2}M^2\right]^{-\frac{\gamma}{\gamma-1}}$$

$$\rho = \rho_0\left[1+\frac{(\gamma-1)}{2}M^2\right]^{-\frac{1}{\gamma-1}}$$

$$\rho_0 = \frac{p_0}{RT_0} \qquad R = \frac{\bar{R}}{\bar{M}}$$

$$u = Ma = M\sqrt{\gamma RT}$$



- perfect gas
- no friction, etc
- isentropic
- quasi-one-dimensional
- constant specific heat
- constant γ
- constant molecular weight
- large (A/A^*) ratios for large Mach numbers

For any given area (A/A^*), the flow can be either subsonic or supersonic

$$T = T_0\left[1+\frac{(\gamma-1)}{2}M^2\right]^{-1}$$

$$p = p_0\left[1+\frac{(\gamma-1)}{2}M^2\right]^{-\frac{\gamma}{\gamma-1}}$$

$$\rho = \rho_0\left[1+\frac{(\gamma-1)}{2}M^2\right]^{-\frac{1}{\gamma-1}}$$

$$\rho_0 = \frac{p_0}{RT_0} \qquad R = \frac{\bar{R}}{\bar{M}}$$

$$u = Ma = M\sqrt{\gamma RT}$$

Courtesy of Professor Gruntman

Nozzle Flow

Use to determine Throat Area

$$A^* = \frac{\dot{m}}{P_0}\sqrt{\frac{T_0 R}{\gamma}}\left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

Assuming Optimum Expansion at sea level

$$M_e = \sqrt{\frac{2\left(\left(\frac{p_e}{p_0}\right)^{-\frac{\gamma-1}{\gamma}} - 1\right)}{\gamma - 1}}$$

Chamber Sizing

We now know:

- Nozzle Throat Area
- Nozzle Exit Area

But how big should the combustion chamber be?

Chamber Volume

- **Traditional Chamber Sizing Method:**
- **Use Characteristic Chamber Length, L* (L-star)**

L-star

Chamber Volume

$$\boxed{L^* = \frac{V_{\mathrm{ch}}}{A^*}}$$

Throat Area

- **Review L-star for past/ current engines which:**
  - Are in the same size-class as engine to be designed
  - Use the same propellants as the engine to be designed

Chamber Geometry

$$V_{\mathrm{ch}} = L^{*} A^{*}$$

Chamber Volume

Chamber Diameter

Chamber Length

Table 1: Chamber Characteristic Length, L*

Propellant Combination	L*, cm
Nitric acid/hydrazine-base fuel	76-89
Nitrogen tetroxide/hydrazine-base fuel	76-89
Hydrogen peroxide/RP-1 (including catalyst bed)	152-178
Liquid oxygen/RP-1	102-127
Liquid oxygen/ammonia	76-102
Liquid oxygen/liquid hydrogen (GH_2 injection)	56-71
Liquid oxygen/liquid hydrogen (LH_2 injection)	76-102
Liquid fluorine/liquid hydrogen (GH_2 injection)	56-66
Liquid fluorine/liquid hydrogen (LH_2 injection)	64-76
Liquid fluorine/hydrazine	61-71
Chlorine trifluoride/hydrazine-base fuel	51-89

From http://www.braeunig.us/space/propuls.htm#engine

Chamber Geometry

Chamber diameter & length:

Long/ narrow chamber → Faster gas flow → More pressure losses due to friction
→ More heat transfer to chamber walls
→ More surface area to cool (bad)

Short/ fat chamber → More hoop stress → Thicker/ heavier walls

Engine Design Cheat-Sheet

$$F_T = \dot{m}\, u_e + A_e \left(p_e - p_a\right)$$

$$u_e = \sqrt{2 \frac{\bar{R}\gamma}{\gamma - 1} \frac{T_0}{\bar{M}} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$V_{ch} = L^* A^*$$

$$T = T_0\left[1 + \frac{(\gamma-1)}{2}M^2\right]^{-1}$$

$$p = p_0\left[1 + \frac{(\gamma-1)}{2}M^2\right]^{-\frac{\gamma}{\gamma-1}}$$

$$\rho = \rho_0\left[1 + \frac{(\gamma-1)}{2}M^2\right]^{-\frac{1}{\gamma-1}}$$

$$\rho_0 = \frac{p_0}{RT_0} \qquad R = \frac{\bar{R}}{\bar{M}}$$

$$u = Ma = M\sqrt{\gamma RT}$$

$$A^* = \frac{\dot{m}}{P_0}\sqrt{\frac{T_0 R}{\gamma}}\left(1 + \frac{\gamma-1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$M_e = \sqrt{\frac{2\left(\left(\frac{p_e}{p_0}\right)^{-\frac{\gamma-1}{\gamma}} - 1\right)}{\gamma - 1}}$$

J&J Design & Analysis

Engine & Injector Sizing

Single Engine Design Point

Design Point J&J	Thermochemistry
$\dot{M}_{TOT} = 1.15\ kg/s$	From NASA CEA
OF ratio= 1.875	Chemistry: Kerosene/Gaseous Oxygen (GOX)
$P_c = 6.895\ MPa, (1000\ psi, 69\ bars)$	$T_c = 3266\ K, (5418\ °F)$
$P_e = 101352.9\ Pa\ (14.7\ psi, 1.01325\ bars)$	$\bar{M} = 20.05\ kg/kmol$
$L^* = 1.27\ m, (50\ inches)$	$\gamma = 1.187$

J&J Design & Analysis

Engine & Injector Sizing

Single Engine

Propellant Mass Flow Rates

$$\frac{\dot{m}_o}{\dot{m}_F} = 1.875$$

$$\dot{m}_o + \dot{m}_F = 1.15\ kg/s$$

$$\dot{m}_F = 1.15 - \dot{m}_o \quad \frac{\dot{m}_o}{1.15 - \dot{m}_o}\ 1.875$$

$$\dot{m}_o = 1.875(1.15 - \dot{m}_o)$$

$$\boldsymbol{\dot{m}_F = 0.4\ \mathrm{kg/s}}$$

$$\boldsymbol{\dot{m}_o = 0.75\ \mathrm{kg/s}}$$

Throat Area

$$A^* = \frac{\dot{M}_{TOT}}{P_0}\sqrt{\frac{T_0 R}{\gamma}}(1 + \frac{\gamma - 1}{2})^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$A^* = \frac{1.15}{6.895\ MPa}\sqrt{\frac{(3265.5)(414.66)}{1.187}}(1 + \frac{1.187 - 1}{2})^{\frac{1.187+1}{2(1.187-1)}}$$

$$\boldsymbol{A^* = 300.4\ mm^2\ , (0.466\ inch^2)}$$

J&J Design & Analysis

Engine & Injector Sizing

Single Engine

Throat Diameter

$$D^* = 2\left(\frac{A^*}{\pi}\right)^{0.5}$$

$$D^* = (2)\left(\frac{3E-4}{\pi}\right)^{0.5}$$

$\boldsymbol{D^*}$ **= 0.0195 m (0.770 inch)**

Exit Mach Number

$$M_e = \sqrt{\frac{2\left(\left(\frac{p_e}{p_0}\right)^{-\frac{\gamma-1}{\gamma}}-1\right)}{\gamma-1}}$$

$$M_e = \sqrt{\frac{2\left(\left(\frac{101352.9}{6.895\ MPa}\right)^{-\frac{1.187-1}{1.187}}-1\right)}{1.187-1}}$$

$$\boldsymbol{M_e = 3.178}$$

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Single Engine

Exit to Throat Area Ratio

$$\frac{A_e}{A^*} = \frac{1}{M}[\frac{2}{\gamma+1}(1+\frac{\gamma-1}{2}M^2)]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{A_e}{A^*} = \frac{1}{3.178}[\frac{2}{1.187+1}(1+\frac{1.187-1}{2}(3.178)^2)]^{\frac{1.187+1}{2(1.187-1)}}$$

$$\boldsymbol{\frac{A_e}{A^*} = 9.1041}$$

Exit Velocity

$$u_e = \sqrt{2\frac{\bar{R}\gamma}{\gamma-1}\frac{T_0}{M}[1-(\frac{p_e}{p_0})^{\frac{\gamma-1}{\gamma}}]}$$

$$u_e = \sqrt{2\frac{(8314)\,(1.187)}{1.187-1}\frac{3265.5}{20.05}[1-(\frac{101352.9}{6.895\ MPa})^{\frac{1.187-1}{1.187}}]}$$

$$\boldsymbol{u_e = 2889.31\ m/s\ ,\ (6464.8\ mph)}$$

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Specific Impulse

$$Isp = \frac{u_{eq}}{g}$$

$$Isp = \frac{2889.311}{9.8}$$

$$\boldsymbol{Isp = 294.5\ sec}$$

Thrust

$$F_T = \dot{m}u_e + A_e(p_e - p_a)$$

$$F_T = (1.15)(2889.31) + 0.0027(101352.9 - 6.895 * 10^6)$$

$\boldsymbol{F_T}$ **= 3.32 kN (747 lbf)**

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Single Engine

Chamber Volume

$$V_{ch} = L^* A^*$$

$$V_{ch} = (1.27)(3.004 * 10^{-4})$$

$$\boldsymbol{V_{ch} = 381.5\ cm^3, (23.28\ inch^3)}$$

Propellants	Characteristic Length (L^*)	
	Low (m)	High (m)
Liquid fluorine / hydrazine	0.61	0.71
Liquid fluorine / gaseous H_2	0.56	0.66
Liquid fluorine / liquid H_2	0.64	0.76
Nitric acid / hydrazine	0.76	0.89
N_2O_4 / hydrazine	0.60	0.89
Liquid O_2 / ammonia	0.76	1.02
Liquid O_2 / gaseous H_2	0.56	0.71
Liquid O_2 / liquid H_2	0.76	1.02
Liquid O_2 / RP-1	1.02	1.27
H_2O_2 / RP-1 (including catalyst)	1.52	1.78

How to determine characteristic length

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Single Engine

Chamber Length

$\boldsymbol{A_t = 3E-4\ m^2}$, $(\boldsymbol{0.466\ inch^2})$

$\boldsymbol{D_t = 1.96\ cm}$, $(\boldsymbol{0.77\ inch})$

$$\frac{A_c}{A_t} = 8D_t^{-0.6} + 1.25$$

$$\frac{A_c}{A_t} = (8)1.96^{-0.6} + 1.25$$

$$\frac{A_c}{A_t} = 6.59$$

$$A_c = 0.002\ \ m^2, (3.10\ inch^2)$$

$$L_C = \frac{V_c}{A_c}$$

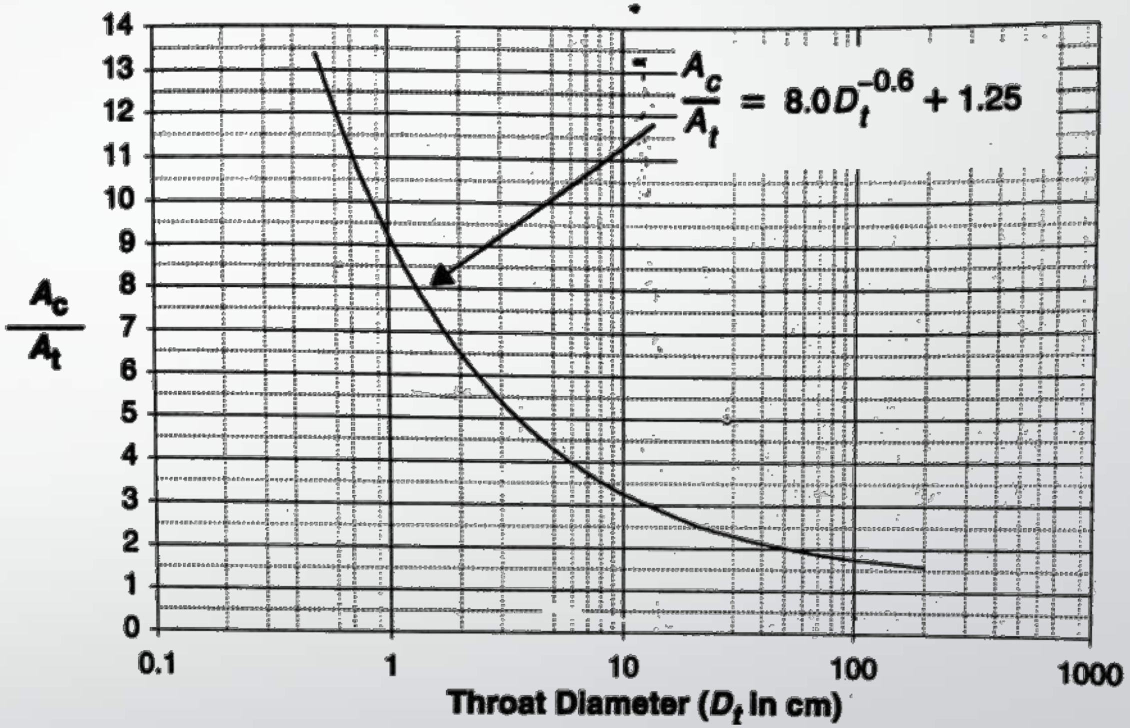
$$L_C = \frac{3.815\ E-4\ m^3}{0.002\ m^2}$$

$$L_C = 0.19\ m\ (7.51\ inch)$$

Use as a starting point. Ended with:

$$\boldsymbol{L_C = 0.17\ m\ (6.58\ inch)}$$

$$\boldsymbol{D_C = 54\ mm\ (2.125\ inch)}$$



Chamber Sizing

Chamber Book "spaghetti"

Chamber Volume 23.28 $inch^3$

Chamber Length: ***7.51"***

Chamber Diameter: ***1.96"***

Chamber "Pancake"

Chamber Volume 23.28 $inch^3$

Chamber Diameter: **4"**

Chamber Length: **2.43"**

Chamber Actual

Chamber Volume 23.28 $inch^3$

Chamber Diameter: **6.58"**

Chamber Length: **2.125"**

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Engine & Injector Sizing

Single Engine

Nozzle Length (Conical)

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$$L_n = \frac{D_e - D_t}{2tan\theta_{cn}}$$

Where L_n=conical nozzle length

D_t=nozzle throat diameter

θ_{cn}= nozzle cone half angle (15°)

$$L_n = \frac{0.059 - 0.02}{2\tan(15°)}$$

$$\boldsymbol{L_n = 2.87\ in\ (72.8\ mm)}$$

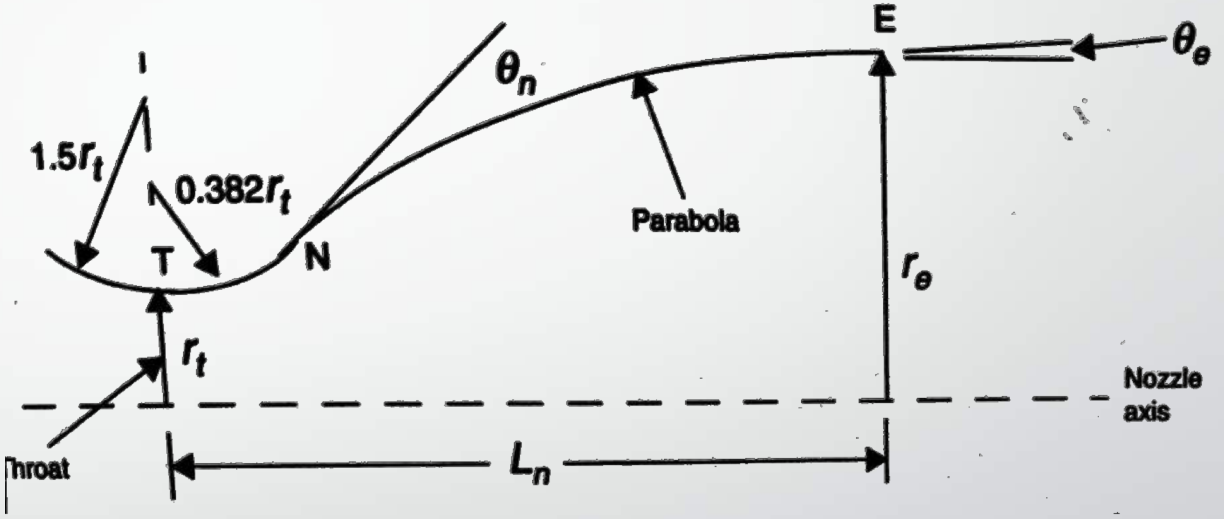

Diagram is for a parabola shaped nozzle. J&J used this diagram for sizing the converging & diverging part of the nozzle

LPL

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Summary of Engine Specifications
Single Engine Static Fire

Propellant	Kerosene	Gaseous Oxygen
OF ratio	1.875	
$\dot{M}_{TOT}$	$1.15\ kg/s$	$2.5\ lbm/s$
P_c	$6.895\ MPa$	$1000\ psi$
P_e	$101352.9\ Pa$	$14.7\ psi$
L^*	$1.27\ m$	$50\ inches$
D^*	19.6 mm	0.770 inch
T_c	$3266\ K$	5418 °F
A^*	$0.3004\ \mathrm{mm}^2$	$0.466\ \mathrm{inch}^2$
A/A^*	9.1041	
Isp	$294.5\ s$	
F_T	3.32 kN	$750\ lbf$
V_{ch}	$381.5\ cm^3$	$23.286\ inch^3$
L_c	$0.17\ m$	$6.58\ inch$
D_c	$54\ mm$	$2.125\ inch$
L_n	$72.8\ mm$	$2.87\ inch$
T_w	$3.81\ mm$	$0.15\ inch$

L P L

Thanks!